PITAGORAS
ACADEMIA

TRIGONOMETRÍA

Profesor
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IDENTIDADES TRIGONOMÉTRICAS

IDENTIDADES TRIGONOMÉTRICAS FUNDAMENTALES

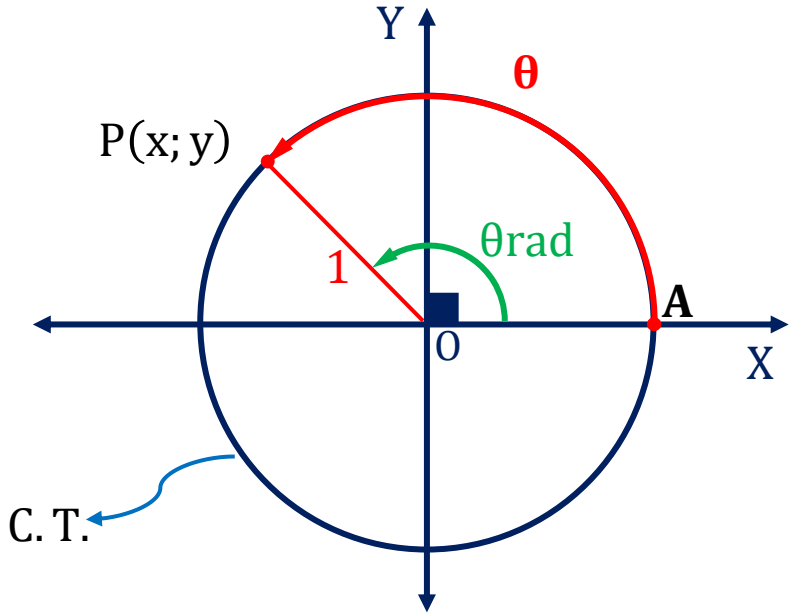
PITAGORAS ACADEMIA

IDENTIDADES TRIGONOMÉTRICAS FUNDAMENTALES

1. IDENTIDADES POR COCIENTE :

- $\text{Tanx} = \dfrac{\text{Senx}}{\text{Cosx}} \quad \forall x \epsilon \mathbb{R} - (2n+1)\dfrac{\pi}{2}; n \in \mathbb{Z}$

- $\text{Cotx} = \dfrac{\text{Cosx}}{\text{Senx}} \quad \forall x \epsilon \mathbb{R} - n\pi; n \in \mathbb{Z}$



$$\text{Sen}(\theta) = \text{Sen}(\theta\text{rad}) = \frac{y}{r} = \frac{y}{1} = y$$

$$\text{Cos}(\theta) = \text{Cos}(\theta\text{rad}) = \frac{x}{r} = \frac{x}{1} = x$$

$$\text{Tan}(\theta) = \text{Tan}(\theta\text{rad}) = \frac{y}{x} = \frac{\text{Sen}\theta}{\text{Cos}\theta}$$

$$\text{Cot}(\theta) = \text{Cot}(\theta\text{rad}) = \frac{x}{y} = \frac{\text{Cos}\theta}{\text{Sen}\theta}$$

IDENTIDADES TRIGONOMÉTRICAS FUNDAMENTALES

2. IDENTIDADES RECÍPROCAS:

- $\text{Senx}.\text{Cscx} = 1$
  $\forall x \epsilon \mathbb{R} - n\pi\,; n \in \mathbb{Z}$

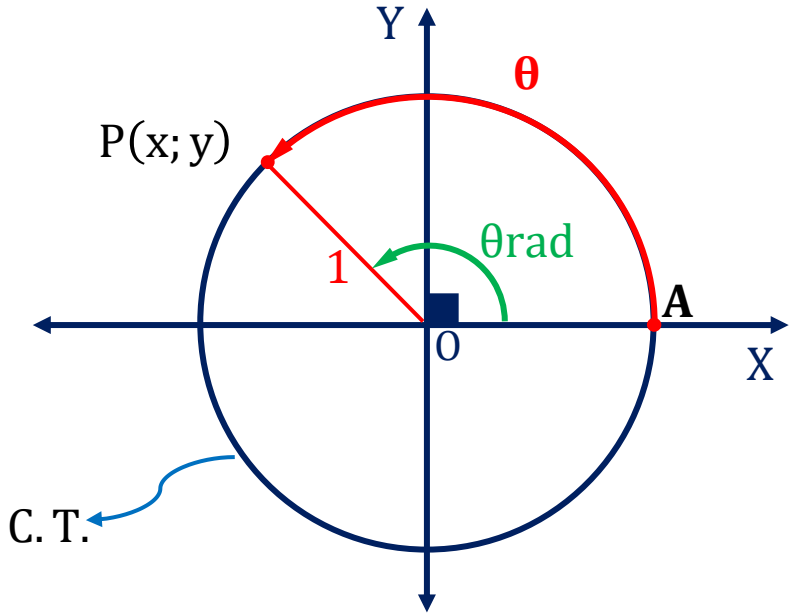
$$\text{Senx} = \frac{1}{\text{Cscx}} \qquad \text{Cscx} = \frac{1}{\text{Senx}}$$

- $\text{Cosx}.\text{Secx} = 1$
  $\forall x \epsilon \mathbb{R} - (2n+1)\frac{\pi}{2}\,; n \in \mathbb{Z}$

$$\text{Cosx} = \frac{1}{\text{Secx}} \qquad \text{Secx} = \frac{1}{\text{Cosx}}$$

- $\text{Tanx}.\text{Cotx} = 1$
  $\forall x \epsilon \mathbb{R} - \frac{n\pi}{2}\,; n \in \mathbb{Z}$

$$\text{Tanx} = \frac{1}{\text{Cotx}} \qquad \text{Cotx} = \frac{1}{\text{Tanx}}$$



$$\text{Sen}(\theta) = \text{Sen}(\theta\text{rad}) = \frac{y}{r} = y$$

$$\text{Cos}(\theta) = \text{Cos}(\theta\text{rad}) = \frac{x}{r} = x$$

$$\vdots \qquad \vdots \qquad \vdots$$

$$\text{Sec}(\theta) = \text{Sec}(\theta\text{rad}) = \frac{r}{x} = \frac{1}{\text{Cos}\theta}$$

$$\text{Csc}(\theta) = \text{Csc}(\theta\text{rad}) = \frac{r}{y} = \frac{1}{\text{Sen}\theta}$$

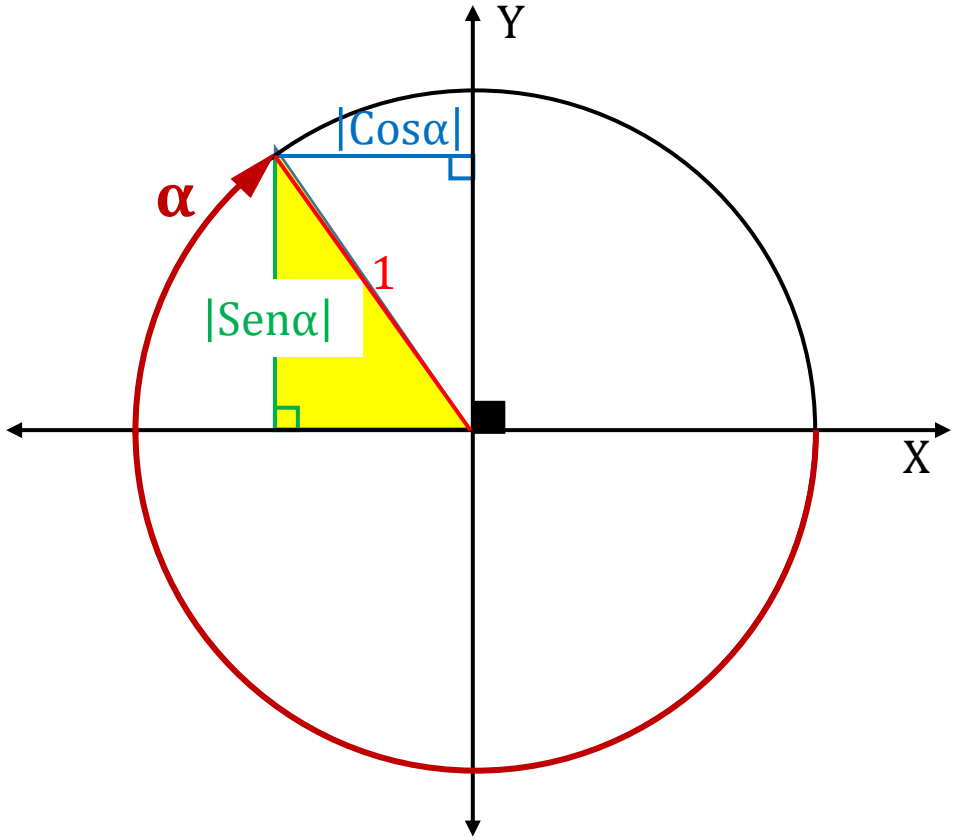
IDENTIDADES TRIGONOMÉTRICAS FUNDAMENTALES

3. IDENTIDADES PITAGÓRICAS :

- $\text{Sen}^2x + \text{Cos}^2x = 1$
  $\forall x \in \mathbb{R}$

  $\text{Sen}^2x = 1 - \text{Cos}^2x$

  $\text{Cos}^2x = 1 - \text{Sen}^2x$



- Por el Teorema de Pitágoras:

$$1^2 = (|\text{Sen}\alpha|)^2 + (|\text{Cos}\alpha|)^2$$
$$1 = \text{Sen}^2\alpha + \text{Cos}^2\alpha$$

IDENTIDADES TRIGONOMÉTRICAS FUNDAMENTALES

3. IDENTIDADES PITAGÓRICAS :

- $1 + \text{Tan}^2x = \text{Sec}^2x$

$\forall x \in \mathbb{R} - (2n+1)\frac{\pi}{2}; n \in \mathbb{Z}$

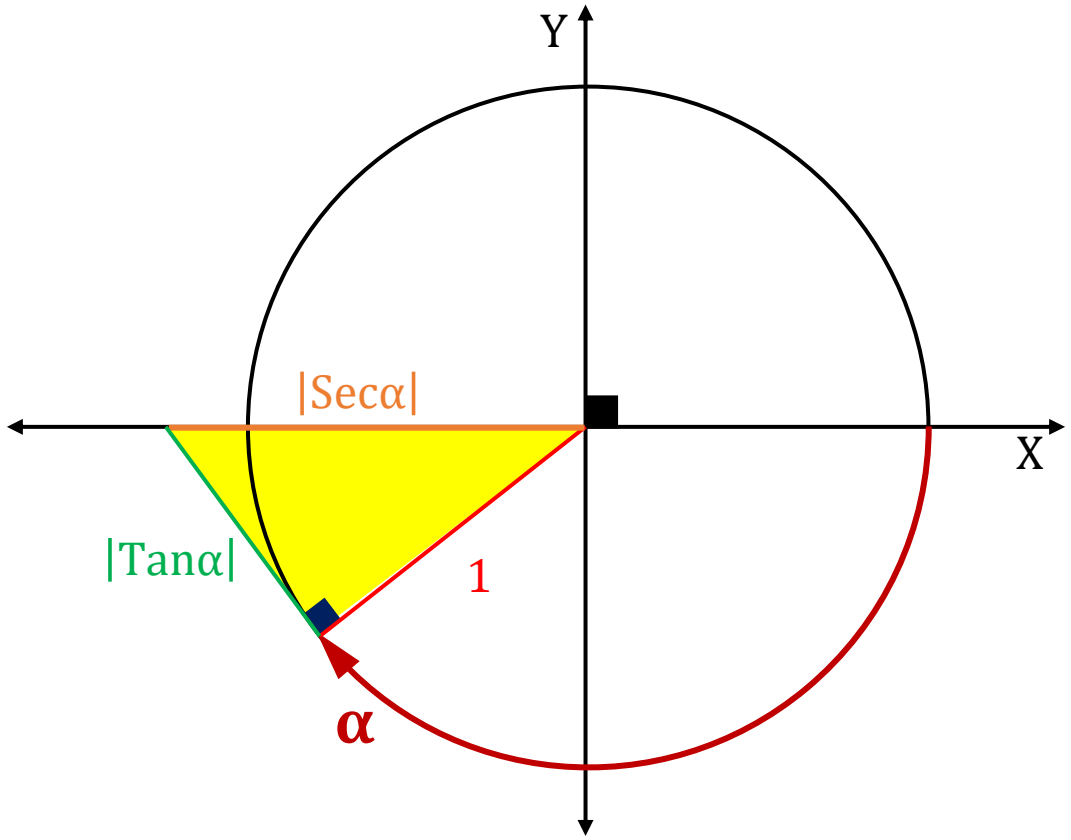
$\text{Tan}^2x = \text{Sec}^2x - 1$

$\text{Sec}^2x - \text{Tan}^2x = 1$

- Por el Teorema de Pitágoras:

$$(|\text{Sec}\alpha|)^2 = 1 + (|\text{Tan}\alpha|)^2$$

$$\text{Sec}^2\alpha = 1 + \text{Tan}^2\alpha$$



IDENTIDADES TRIGONOMÉTRICAS FUNDAMENTALES

3. IDENTIDADES PITAGÓRICAS :

- $1 + \text{Cot}^2 x = \text{Csc}^2 x$

  $\forall x \epsilon \mathbb{R} - n\pi \, ; n \in \mathbb{Z}$

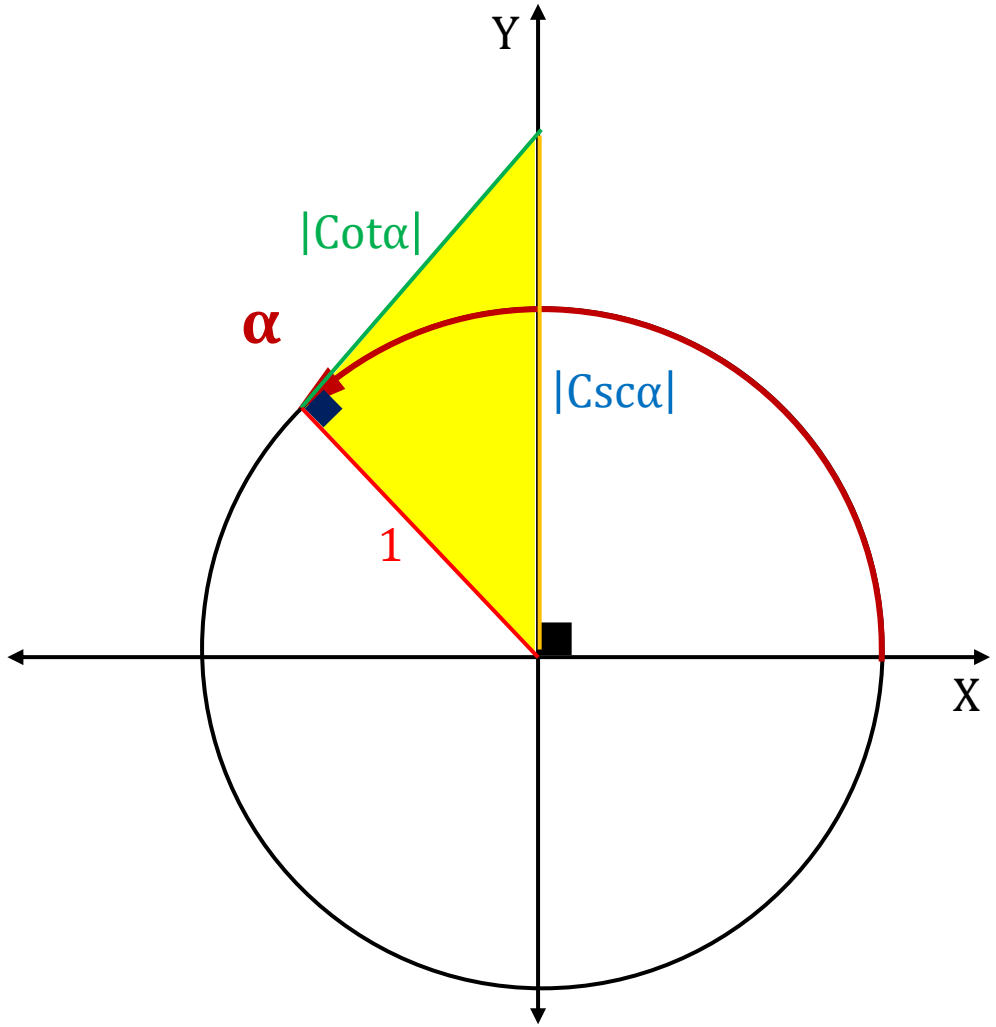
  $\text{Cot}^2 x = \text{Csc}^2 x - 1$

  $\text{Csc}^2 x - \text{Cot}^2 x = 1$

- Por el Teorema de Pitágoras:

$$(|\text{Csc}\alpha|)^2 = 1 + (|\text{Cot}\alpha|)^2$$

$$\text{Csc}^2\alpha = 1 + \text{Cot}^2\alpha$$



PITAGORAS ACADEMIA

IDENTIDADES TRIGONOMÉTRICAS FUNDAMENTALES

IDENTIDADES PITAGÓRICAS :

- $\text{Sen}^2x + \text{Cos}^2x = 1$ $\quad \forall x \epsilon \mathbb{R}$
  - $\text{Sen}^2x = 1 - \text{Cos}^2x$
  - $\text{Cos}^2x = 1 - \text{Sen}^2x$
- $1 + \text{Tan}^2x = \text{Sec}^2x$ $\quad \forall x \epsilon \mathbb{R} - (2n+1)\frac{\pi}{2}; n \in \mathbb{Z}$
  - $\text{Tan}^2x = \text{Sec}^2x - 1$
  - $\text{Sec}^2x - \text{Tan}^2x = 1$
- $1 + \text{Cot}^2x = \text{Csc}^2x$ $\quad \forall x \epsilon \mathbb{R} - n\pi ; n \in \mathbb{Z}$
  - $\text{Cot}^2x = \text{Csc}^2x - 1$
  - $\text{Csc}^2x - \text{Cot}^2x = 1$

IDENTIDADES RECÍPROCAS:

- $\text{Senx}.\text{Cscx} = 1$ $\quad \forall x \epsilon \mathbb{R} - n\pi ; n \in \mathbb{Z}$
  - $\text{Senx} = \frac{1}{\text{Cscx}}$
  - $\text{Cscx} = \frac{1}{\text{Senx}}$
- $\text{Cosx}.\text{Secx} = 1$ $\quad \forall x \epsilon \mathbb{R} - (2n+1)\frac{\pi}{2}; n \in \mathbb{Z}$
  - $\text{Cosx} = \frac{1}{\text{Secx}}$
  - $\text{Secx} = \frac{1}{\text{Cosx}}$
- $\text{Tanx}.\text{Cotx} = 1$ $\quad \forall x \epsilon \mathbb{R} - \frac{n\pi}{2}; n \in \mathbb{Z}$
  - $\text{Tanx} = \frac{1}{\text{Cotx}}$
  - $\text{Cotx} = \frac{1}{\text{Tanx}}$

IDENTIDADES POR COCIENTE :

- $\text{Tanx} = \frac{\text{Senx}}{\text{Cosx}}$ $\quad \forall x \epsilon \mathbb{R} - (2n+1)\frac{\pi}{2}; n \in \mathbb{Z}$
- $\text{Cotx} = \frac{\text{Cosx}}{\text{Senx}}$ $\quad \forall x \epsilon \mathbb{R} - n\pi; n \in \mathbb{Z}$

IDENTIDADES TRIGONOMÉTRICAS FUNDAMENTALES

4. IDENTIDADES AUXILIARES:

a) $(\text{Senx} \pm \text{Cosx})^2 = 1 \pm 2\text{SenxCosx}$

Demostración:

$(\text{Senx} + \text{Cosx})^2 = \underbrace{\text{Sen}^2\text{x} + 2\text{SenxCosx} + \text{Cos}^2\text{x}}_{1}$

$(\text{Senx} + \text{Cosx})^2 = 1 + 2\text{SenxCosx}$

b) $\text{Sen}^4\text{x} + \text{Cos}^4\text{x} = 1 - 2\text{Sen}^2\text{xCos}^2\text{x}$

Demostración:

$\text{Sen}^2\text{x} + \text{Cos}^2\text{x} = 1$

AL^2: $(\text{Sen}^2\text{x} + \text{Cos}^2\text{x})^2 = 1^2$

$\text{Sen}^4\text{x} + 2\text{Sen}^2\text{xCos}^2\text{x} + \text{Cos}^4\text{x} = 1$

$\text{Sen}^4\text{x} + \text{Cos}^4\text{x} = 1 - 2\text{Sen}^2\text{xCos}^2\text{x}$

IDENTIDADES TRIGONOMÉTRICAS FUNDAMENTALES

4. IDENTIDADES AUXILIARES:

c) $\text{Sen}^6x + \text{Cos}^6x = 1 - 3\text{Sen}^2x\text{Cos}^2x$

Demostración:

$\text{Sen}^2x + \text{Cos}^2x = 1$

AL^3: $(\text{Sen}^2x + \text{Cos}^2x)^3 = 1^3$

$$\text{Sen}^6x + 3\text{Sen}^2x\text{Cos}^2x\underbrace{(\text{Sen}^2x + \text{Cos}^2x)}_{1} + \text{Cos}^6 x = 1$$

$$\text{Sen}^6x + \text{Cos}^6 x = 1 - 3\text{Sen}^2x\text{Cos}^2x$$

IDENTIDADES TRIGONOMÉTRICAS FUNDAMENTALES

4. IDENTIDADES AUXILIARES:

d) $\text{Sen}^8x + \text{Cos}^8x = 1 - 4\text{Sen}^2x\text{Cos}^2x + 2\text{Sen}^4x\text{Cos}^4x$

Demostración:

$$\text{Sen}^4x + \text{Cos}^4x = 1 - 2\text{Sen}^2x\text{Cos}^2x$$

AL^2: $(\text{Sen}^4x + \text{Cos}^4x)^2 = (1 - 2\text{Sen}^2x\text{Cos}^2x)^2$

$$\text{Sen}^8x + 2\text{Sen}^4x\text{Cos}^4x + \text{Cos}^8x = 1 - 4\text{Sen}^2x\text{Cos}^2x + 4\text{Sen}^4x\text{Cos}^4x$$

$$\text{Sen}^8x + \text{Cos}^8x = 1 - 4\text{Sen}^2x\text{Cos}^2x + 2\text{Sen}^4x\text{Cos}^4x$$

IDENTIDADES TRIGONOMÉTRICAS FUNDAMENTALES

4. IDENTIDADES AUXILIARES:

e) $\text{Tanx} + \text{Cotx} = \text{Secx}.\text{Cscx}$

Demostración:

$$\text{Tanx} + \text{Cotx} = \frac{\text{Senx}}{\text{Cosx}} + \frac{\text{Cosx}}{\text{Senx}}$$

$$\text{Tanx} + \text{Cotx} = \frac{\overbrace{\text{Sen}^2\text{x} + \text{Cos}^2\text{x}}^{1}}{\text{CosxSenx}}$$

$$\text{Tanx} + \text{Cotx} = \frac{1}{\text{Cosx}} \times \frac{1}{\text{Senx}}$$

$$\text{Tanx} + \text{Cotx} = \text{Secx}.\text{Cscx}$$

IDENTIDADES TRIGONOMÉTRICAS FUNDAMENTALES

4. IDENTIDADES AUXILIARES:

f) $\text{Sec}^2x + \text{Csc}^2x = \text{Sec}^2x.\text{Csc}^2x$

Demostración:

$$\text{Sec}^2x + \text{Csc}^2x = \frac{1}{\text{Cos}^2x} + \frac{1}{\text{Sen}^2x}$$

$$\text{Sec}^2x + \text{Csc}^2x = \frac{\overbrace{\text{Sen}^2x + \text{Cos}^2x}^{1}}{\text{Cos}^2x\text{Sen}^2x}$$

$$\text{Sec}^2x + \text{Csc}^2x = \frac{1}{\text{Cos}^2x} \times \frac{1}{\text{Sen}^2x}$$

$$\text{Sec}^2x + \text{Csc}^2x = \text{Sec}^2x.\text{Csc}^2x$$

IDENTIDADES TRIGONOMÉTRICAS FUNDAMENTALES

4. IDENTIDADES AUXILIARES:

g) $\text{Tan}^2x - \text{Sen}^2x = \text{Tan}^2x.\,\text{Sen}^2x$

Demostración:

$$\text{Tan}^2x - \text{Sen}^2x = \frac{\text{Sen}^2x}{\text{Cos}^2x} - \text{Sen}^2x$$

$$\text{Tan}^2x - \text{Sen}^2x = \text{Sen}^2x\left(\frac{1}{\text{Cos}^2x} - 1\right)$$

$$\text{Tan}^2x - \text{Sen}^2x = \text{Sen}^2x\underbrace{(\text{Sec}^2x - 1)}_{\text{Tan}^2x}$$

$$\text{Tan}^2x - \text{Sen}^2x = \text{Tan}^2x.\,\text{Sen}^2x$$

IDENTIDADES TRIGONOMÉTRICAS FUNDAMENTALES

4. IDENTIDADES AUXILIARES:

h) $\text{Cot}^2x - \text{Cos}^2x = \text{Cot}^2x.\text{Cos}^2x$

Demostración:

$$\text{Cot}^2x - \text{Cos}^2x = \frac{\text{Cos}^2x}{\text{Sen}^2x} - \text{Cos}^2x$$

$$\text{Cot}^2x - \text{Cos}^2x = \text{Cos}^2x\left(\frac{1}{\text{Sen}^2x} - 1\right)$$

$$\text{Cot}^2x - \text{Cos}^2x = \text{Cos}^2x\underbrace{(\text{Csc}^2x - 1)}_{\text{Cot}^2x}$$

$$\text{Cot}^2x - \text{Cos}^2x = \text{Cot}^2x.\text{Cos}^2x$$

IDENTIDADES TRIGONOMÉTRICAS FUNDAMENTALES

4. IDENTIDADES AUXILIARES:

i) $(1 \pm \text{Senx} \pm \text{Cosx})^2 = 2(1 \pm \text{Senx})(1 \pm \text{Cosx})$

Demostración:

$$(1 + \text{Senx} - \text{Cosx})^2 = 1^2 + \underbrace{\text{Sen}^2\text{x} + \text{Cos}^2\text{x}}_{1} + 2\text{Senx} - 2\text{Cosx} - 2\text{Senx.Cosx}$$

$$(1 + \text{Senx} - \text{Cosx})^2 = 2\,(1 + \text{Senx} - \text{Cosx} - \text{Senx.Cosx})$$

$$(1 + \text{Senx} - \text{Cosx})^2 = 2[(1 + \text{Senx}) - \text{Cosx}(1 + \text{Senx})]$$

$$(1 + \text{Senx} - \text{Cosx})^2 = 2(1 + \text{Senx})\,(1 - \text{Cosx})$$

IDENTIDADES TRIGONOMÉTRICAS FUNDAMENTALES

4. IDENTIDADES AUXILIARES:

j) $$\frac{\text{Senx}}{1 \mp \text{Cosx}} = \frac{1 \pm \text{Cosx}}{\text{Senx}}$$

Demostración:

$$\text{Sen}^2\text{x} = 1 - \text{Cos}^2\text{x}$$

$$\text{Senx}.\,\text{Senx} = (1 + \text{Cosx})(1 - \text{Cosx})$$

$$\frac{\text{Senx}}{1 + \text{Cosx}} = \frac{1 - \text{Cosx}}{\text{Senx}}$$

IDENTIDADES TRIGONOMÉTRICAS FUNDAMENTALES

4. IDENTIDADES AUXILIARES:

k) $$\frac{\text{Cosx}}{1 \pm \text{Senx}} = \frac{1 \mp \text{Senx}}{\text{Cosx}}$$

Demostración:

$$\text{Cos}^2\text{x} = 1 - \text{Sen}^2\text{x}$$

$$\text{Cosx}.\,\text{Cosx} = (1 + \text{Senx})(1 - \text{Senx})$$

$$\frac{\text{Cosx}}{1 - \text{Senx}} = \frac{1 + \text{Senx}}{\text{Cosx}}$$

IDENTIDADES TRIGONOMÉTRICAS FUNDAMENTALES

4. IDENTIDADES AUXILIARES:

l) Si: $\text{Secx} + \text{Tanx} = \text{N} \longleftrightarrow \text{Secx} - \text{Tanx} = \frac{1}{\text{N}}$

Demostración:

$$\text{Sec}^2\text{x} - \text{Tan}^2\text{x} = 1$$

$$(\text{Secx} + \text{Tanx})(\text{Secx} - \text{Tanx}) = 1$$

$$\text{Secx} + \text{Tanx} = \frac{1}{\text{Secx} - \text{Tanx}}$$

4. IDENTIDADES AUXILIARES:

m) Si: $\text{Cscx} - \text{Cotx} = \text{M} \longleftrightarrow \text{Cscx} + \text{Cotx} = \frac{1}{\text{M}}$

Demostración:

$$\text{Csc}^2\text{x} - \text{Cot}^2\text{x} = 1$$

$$(\text{Cscx} + \text{Cotx})(\text{Cscx} - \text{Cotx}) = 1$$

$$\text{Cscx} - \text{Cotx} = \frac{1}{\text{Cscx} + \text{Cotx}}$$

IDENTIDADES TRIGONOMÉTRICAS FUNDAMENTALES

4. IDENTIDADES AUXILIARES:

a) $(\text{Senx} \pm \text{Cosx})^2 = 1 \pm 2\text{SenxCosx}$

b) $\text{Sen}^4x + \text{Cos}^4x = 1 - 2\text{Sen}^2x\text{Cos}^2x$

c) $\text{Sen}^6x + \text{Cos}^6x = 1 - 3\text{Sen}^2x\text{Cos}^2x$

d) $\text{Sen}^8x + \text{Cos}^8x = 1 - 4\text{Sen}^2x\text{Cos}^2x + 2\text{Sen}^4x\text{Cos}^4x$

e) $\text{Tanx} + \text{Cotx} = \text{Secx}.\text{Cscx}$

f) $\text{Sec}^2x + \text{Csc}^2x = \text{Sec}^2x.\text{Csc}^2x$

g) $\text{Tan}^2x - \text{Sen}^2x = \text{Tan}^2x.\text{Sen}^2x$

h) $\text{Cot}^2x - \text{Cos}^2x = \text{Cot}^2x.\text{Cos}^2x$

i) $(1 \pm \text{Senx} \pm \text{Cosx})^2 = 2(1 \pm \text{Senx})(1 \pm \text{Cosx})$

j) $\dfrac{\text{Senx}}{1 \mp \text{Cosx}} = \dfrac{1 \pm \text{Cosx}}{\text{Senx}}$

k) $\dfrac{\text{Cosx}}{1 \pm \text{Senx}} = \dfrac{1 \mp \text{Senx}}{\text{Cosx}}$

l) Si: $\text{Secx} + \text{Tanx} = \text{N} \longleftrightarrow \text{Secx} - \text{Tanx} = \dfrac{1}{\text{N}}$

m) Si: $\text{Cscx} - \text{Cotx} = \text{M} \longleftrightarrow \text{Cscx} + \text{Cotx} = \dfrac{1}{\text{M}}$

IDENTIDADES TRIGONOMÉTRICAS FUNDAMENTALES

5. PROPIEDAD:

Si: $aSenx + bCosx = c\,;\, a, b\ y\ c\ \in \mathbb{R}$

además: $a^2 + b^2 = c^2$

entonces:

$$Senx = \frac{a}{c} \quad \wedge \quad Cosx = \frac{b}{c}$$

Demostración:

$$bCosx = c - aSenx$$

AL^2: $(bCosx)^2 = (c - aSenx)^2$

$$b^2Cos^2x = c^2 - 2acSenx + a^2Sen^2x$$

$$b^2(1 - Sen^2x) = c^2 - 2acSenx + a^2Sen^2x$$

$$b^2 - b^2Sen^2x = c^2 - 2acSenx + a^2Sen^2x$$

$$\underbrace{(a^2 + b^2)}_{c^2}Sen^2x - 2acSenx + \underbrace{c^2 - b^2}_{a^2} = 0$$

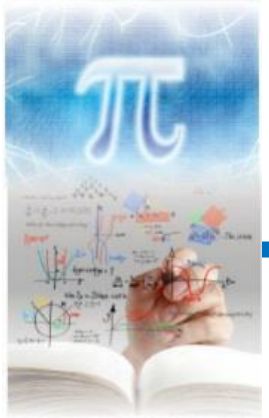
$$c^2Sen^2x - 2acSenx + a^2 = 0$$

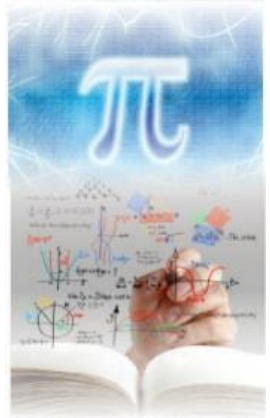
$$\underbrace{(cSenx - a)^2}_{0} = 0$$

$$\longrightarrow Senx = \frac{a}{c}$$

MOMENTO DE PRACTICAR

PROBLEMAS Y RESOLUCIÓN





IDENTIDADES TRIGONOMÉTRICAS FUNDAMENTALES

1.Hallar el equivalente de: $M = \dfrac{Covx(1+Senx)+Versx(1+Cosx)}{Exsecx(Secx+1)}$

RESOLUCIÓN:

$$M = \frac{Covx(1+Senx)+Versx(1+Cosx)}{Exsecx(Secx+1)}$$

$$M = \frac{(1-Senx)(1+Senx)+(1-Cosx)(1+Cosx)}{(Secx-1)(Secx+1)}$$

$$M = \frac{(1-Sen^2x)+(1-Cos^2x)}{Sec^2x-1}$$

$$M = \frac{2-(Cos^2x+Sen^2x)}{Tan^2x}$$

$$M = \frac{1}{Tan^2x}$$

$$\therefore \mathbf{M = Cot^2x}$$

CLAVE: D

IDENTIDADES TRIGONOMÉTRICAS FUNDAMENTALES

2.Simplificar : $P = \dfrac{\text{Tan}^4\theta + \text{Sen}^4\theta - \text{Tan}^4\theta\text{Sen}^4\theta}{(\text{Tan}\theta + \text{Sen}\theta)(\text{Tan}\theta - \text{Sen}\theta)}$

RESOLUCIÓN:

$$P = \frac{\text{Tan}^4\theta + \text{Sen}^4\theta - \text{Tan}^4\theta\text{Sen}^4\theta}{(\text{Tan}\theta + \text{Sen}\theta)(\text{Tan}\theta - \text{Sen}\theta)}$$

$$P = \frac{\text{Tan}^4\theta + \text{Sen}^4\theta - \text{Tan}^4\theta\text{Sen}^4\theta}{\text{Tan}^2\theta - \text{Sen}^2\theta}$$

$$P = \frac{\text{Tan}^4\theta + \text{Sen}^4\theta - \text{Tan}^4\theta\text{Sen}^4\theta}{\text{Tan}^2\theta\text{Sen}^2\theta}$$

$$P = \frac{\mathbf{2\text{Tan}^2\theta\text{Sen}^2\theta}}{\text{Tan}^2\theta\text{Sen}^2\theta}$$

$\therefore \mathbf{P = 2}$

$$\text{Tan}^2\theta - \text{Sen}^2\theta = \text{Tan}^2\theta\text{Sen}^2\theta$$

AL^2: $\text{Tan}^4\theta - 2\text{Tan}^2\theta\text{Sen}^2\theta + \text{Sen}^4\theta = \text{Tan}^4\theta\text{Sen}^4\theta$

$$\mathbf{\text{Tan}^4\theta + \text{Sen}^4\theta - \text{Tan}^4\theta\text{Sen}^4\theta = 2\text{Tan}^2\theta\text{Sen}^2\theta}$$

CLAVE: B

IDENTIDADES TRIGONOMÉTRICAS FUNDAMENTALES

3.Simplificar: $P = \frac{Tan^2x + Cot^2x + 1}{Tanx + Cotx - 1} + \frac{Tan^2x + Cot^2x - 3}{Tanx - Cotx + 1}$

RESOLUCIÓN:

$$P = \frac{Tan^2x + Cot^2x + 1}{Tanx + Cotx - 1} + \frac{Tan^2x + Cot^2x - 3}{Tanx - Cotx + 1}$$

$$P = \frac{a^2 - 2 + 1}{a - 1} + \frac{b^2 + 2 - 3}{b + 1}$$

$$P = \frac{a^2 - 1}{a - 1} + \frac{b^2 - 1}{b + 1}$$

$$P = \frac{(a+1)(a-1)}{(a-1)} + \frac{(b+1)(b-1)}{(b+1)}$$

$$P = a + b$$

$\therefore P = 2Tanx$

Sea: $Tanx + Cotx = a$

AL^2: $Tan^2x + 2TanxCotx + Cot^2x = a^2$

$$Tan^2x + 2 + Cot^2x = a^2$$

$$Tan^2x + Cot^2x = a^2 - 2$$

Sea: $Tanx - Cotx = b$

AL^2: $Tan^2x - 2TanxCotx + Cot^2x = b^2$

$$Tan^2x - 2 + Cot^2x = b^2$$

$$Tan^2x + Cot^2x = b^2 + 2$$

CLAVE: A

IDENTIDADES TRIGONOMÉTRICAS FUNDAMENTALES

4. Si $x \in \left\langle \frac{5\pi}{4}; \frac{3\pi}{2} \right\rangle$, reducir la expresión : $M = \sqrt{1 + 2SenxCosx} - \sqrt{1 - 2SenxCosx}$

RESOLUCIÓN:

$$M = \sqrt{1 + 2SenxCosx} - \sqrt{1 - 2SenxCosx}$$

$$M = \sqrt{(Senx + Cosx)^2} - \sqrt{(Senx - Cosx)^2}$$

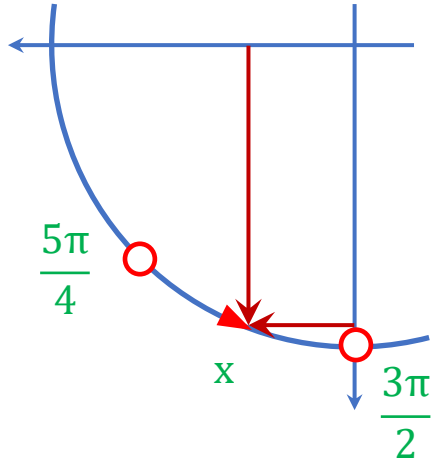
$$M = |Senx + Cosx| - |Senx - Cosx|$$

$$(-) \qquad (-)$$

$$M = (-Senx - Cosx) - (-Senx + Cosx)$$

$$M = -Senx - Cosx + Senx - Cosx$$

$$\therefore \mathbf{M = -2Cosx}$$



$Senx < Cosx$

$Senx - Cosx < 0$

CLAVE: D

IDENTIDADES TRIGONOMÉTRICAS FUNDAMENTALES

5.Si : Tanx - Cscx = 1, obtener : $M = Sen^3x + Cos^3x$

RESOLUCIÓN:

$$Tanx - Cscx = 1$$

$$\frac{Senx}{Cosx} - \frac{1}{Senx} = 1$$

$$\frac{Sen^2x - Cosx}{CosxSenx} = 1$$

$$Sen^2x - Cosx = CosxSenx$$

$$Sen^2x = Cos^2x\frac{(1 + Senx)}{Cosx}$$

$$Sen^2x = Cos^2x\frac{Cosx}{(1 - Senx)}$$

$$Sen^2x(1 - Senx) = Cos^3x$$

$$Sen^2x - Sen^3x = Cos^3x$$

$$\therefore \mathbf{Sen^3x + Cos^3x = Sen^2x}$$

CLAVE: A

IDENTIDADES TRIGONOMÉTRICAS FUNDAMENTALES

6. Si se cumple que : $\frac{1+\text{Vers}^2x+\text{Cov}^2x}{\text{Versx}+\text{Covx}} = 2\text{Versx.Covx}$, calcular el valor de : $M = \text{Secx} + \text{Cscx}$

RESOLUCIÓN:

$$\frac{1+(1-\text{Cosx})^2+(1-\text{Senx})^2}{1-\text{Cosx}+1-\text{Senx}} = 2(1-\text{Cosx})(1-\text{Senx})$$

$$\frac{1+(1-2\text{Cosx}+\text{Cos}^2x)+(1-2\text{Senx}+\text{Sen}^2x)}{2-\text{Cosx}-\text{Senx}} = 2(1-\text{Cosx})(1-\text{Senx})$$

$$\frac{4-2\text{Cosx}-2\text{Senx}}{2-\text{Cosx}-\text{Senx}} = 2(1-\text{Cosx})(1-\text{Senx})$$

$$\frac{2(2-\text{Cosx}-\text{Senx})}{2-\text{Cosx}-\text{Senx}} = 2(1-\text{Cosx})(1-\text{Senx})$$

$$1 = 1-\text{Cosx}-\text{Senx}+\text{SenxCosx}$$

$$\text{Cosx}+\text{Senx} = \text{SenxCosx}$$

$$\frac{\text{Cosx}+\text{Senx}}{\text{SenxCosx}} = 1 \qquad \frac{\text{Cosx}}{\text{SenxCosx}} + \frac{\text{Senx}}{\text{SenxCosx}} = 1$$

$\therefore \mathbf{Cscx + Secx = 1}$

CLAVE: D

IDENTIDADES TRIGONOMÉTRICAS FUNDAMENTALES

7. Si : $\text{Sen}\theta + \text{Tan}\theta + \text{Sec}\theta = 3$, calcular $M = \sqrt{\text{Cos}\theta} + \sqrt{\text{Sen}\theta}.\sqrt{\text{Tan}\theta} + \sqrt{\text{Sec}\theta}$

RESOLUCIÓN:

$$M = \sqrt{\text{Cos}\theta} + \sqrt{\text{Sen}\theta\text{Tan}\theta} + \sqrt{\text{Sec}\theta}$$

$$M^2 = \left(\sqrt{\text{Cos}\theta}\right)^2 + \left(\sqrt{\text{Sen}\theta\text{Tan}\theta}\right)^2 + \left(\sqrt{\text{Sec}\theta}\right)^2 + 2\sqrt{\text{Sen}\theta\text{Cos}\theta\text{Tan}\theta} + 2\sqrt{\text{Sen}\theta\text{Tan}\theta\text{Sec}\theta} + 2\sqrt{\text{Cos}\theta\text{Sec}\theta}$$

$$M^2 = \text{Cos}\theta + \underbrace{\text{Sen}\theta\text{Tan}\theta}_{\text{Sec}\theta - \text{Cos}\theta} + \text{Sec}\theta + 2\sqrt{\text{Sen}^2\theta} + 2\sqrt{\text{Tan}^2\theta} + 2\sqrt{1}$$

$$M^2 = \cancel{\text{Cos}\theta} + \text{Sec}\theta - \cancel{\text{Cos}\theta} + \text{Sec}\theta + 2\text{Sen}\theta + 2\text{Tan}\theta + 2$$

$$M^2 = \underbrace{2\text{Sec}\theta + 2\text{Sen}\theta + 2\text{Tan}\theta}_{6} + 2$$

$$M^2 = 8$$

$$\therefore \mathbf{M = 2\sqrt{2}}$$

CLAVE: B

IDENTIDADES TRIGONOMÉTRICAS FUNDAMENTALES

8. Si : $a = \cot^4 x + \cos^4 x$ y $b = \tan^4 x + \sen^4 x$, obtener: $M = \sen^2 x + \cos^4 x$

RESOLUCIÓN:

$a = \cot^4 x + \cos^4 x$

$a = \cos^4 x(\csc^4 x + 1)$

$b = \tan^4 x + \sen^4 x$

$b = \sen^4 x(\sec^4 x + 1)$

$\times$

$M = \sen^2 x + \cos^4 x$

$M = 1 - \cos^2 x + \cos^4 x$

$M = 1 - \cos^2 x(1 - \cos^2 x)$

$M = 1 - \cos^2 x(\sen^2 x)$

$ab = \sen^4 x(\csc^4 x + 1)\cos^4 x(\sec^4 x + 1)$

$ab = (\sen^4 x + 1)(\cos^4 x + 1)$

$ab = \sen^4 x\cos^4 x + \sen^4 x + \cos^4 x + 1$

$ab = \sen^4 x\cos^4 x + 1 - 2\sen^2 x\cos^2 x + 1$

$ab = 1 + \sen^4 x\cos^4 x - 2\sen^2 x\cos^2 x + 1$

$ab = 1 + (1 - \sen^2 x\cos^2 x)^2$

$ab - 1 = (M)^2$

$M = \pm\sqrt{ab - 1}$

$\therefore \mathbf{M = \sqrt{ab - 1}}$

CLAVE: E

IDENTIDADES TRIGONOMÉTRICAS FUNDAMENTALES

9. Si : $\theta \in \left\langle \frac{\pi}{4}; \frac{\pi}{2} \right\rangle ; b > 0$, además : $Sec\theta - Csc\theta = \sqrt{a}$ y $Sen^4\theta + Cos^4\theta = 1 - b^2$ eliminar "θ"

RESOLUCIÓN:

$$Sen^4\theta + Cos^4\theta = 1 - b^2$$

$$1 - 2Sen^2\theta Cos^2\theta = 1 - b^2$$

$$2Sen^2\theta Cos^2\theta = b^2$$

$$\sqrt{2}Sen\theta Cos\theta = b$$

$$\frac{\sqrt{2}}{b} = \frac{1}{Sen\theta Cos\theta}$$

$$Sec\theta Csc\theta = \frac{\sqrt{2}}{b}$$

$$Sec\theta - Csc\theta = \sqrt{a}$$

AL^2: $Sec^2\theta - 2Sec\theta Csc\theta + Csc^2\theta = a$

$$Sec^2\theta Csc^2\theta - 2Sec\theta Csc\theta = a$$

$$\frac{2}{b^2} - 2\frac{\sqrt{2}}{b} = a$$

$$2 - 2\sqrt{2}b = ab^2$$

$$\therefore \boldsymbol{\left(ab + 2\sqrt{2}\right)b = 2}$$

CLAVE: C

IDENTIDADES TRIGONOMÉTRICAS FUNDAMENTALES

10. Eliminar "α" de : $mSen\alpha = Exsec\alpha.Cov\alpha$ y $n = Sen\alpha + Cos\alpha$

RESOLUCIÓN:

$$mSen\alpha = Exsec\alpha.Cov\alpha$$

$$mSen\alpha = (Sec\alpha - 1)(1 - Sen\alpha)$$

$$mSen\alpha = \left(\frac{1}{Cos\alpha} - 1\right)(1 - Sen\alpha)$$

$$mSen\alpha = \left(\frac{1 - Cos\alpha}{Cos\alpha}\right)(1 - Sen\alpha)$$

$$m = \frac{(1 - Cos\alpha)(1 - Sen\alpha)}{Sen\alpha Cos\alpha}$$

$$m = \frac{2(1 - Cos\alpha)(1 - Sen\alpha)}{2Sen\alpha Cos\alpha}$$

$$m = \frac{(1 - Sen\alpha - Cos\alpha)^2}{(Sen\alpha + Cos\alpha)^2 - 1}$$

$$m = \frac{(1 - n)^2}{(n)^2 - 1}$$

$$m = \frac{(n - 1)^{\cancel{2}}}{(n + 1)\cancel{(n - 1)}}$$

$$\mathbf{m(n + 1) = (n - 1)}$$

CLAVE: A

IDENTIDADES TRIGONOMÉTRICAS FUNDAMENTALES

11. Simplificar : $Q = \dfrac{\sqrt{1+\sqrt{1+4Sec^2x.Tan^2x}}}{\sqrt{1+\sqrt{1+4Csc^2x.Cot^2x}}}$

RESOLUCIÓN:

$$Sec^2x - Tan^2x = 1$$

AL^2: $Sec^4x - 2Sec^2x.Tan^2x + Tan^4x = 1$

$$Sec^4x + Tan^4x = 1 + 2Sec^2x.Tan^2x$$

$$Sec^4x + Tan^4x + 2Sec^2x.Tan^2x = 1 + 4Sec^2x.Tan^2x$$

$$(Sec^2x + Tan^2x)^2 = 1 + 4Sec^2x.Tan^2x$$

Análogamente:

$$(Csc^2x + Cot^2x)^2 = 1 + 4Csc^2x.Cot^2x$$

$$Q = \frac{\sqrt{1+\sqrt{1+4Sec^2x.Tan^2x}}}{\sqrt{1+\sqrt{1+4Csc^2x.Cot^2x}}}$$

$$Q = \frac{\sqrt{1+\sqrt{(Sec^2x + Tan^2x)^2}}}{\sqrt{1+\sqrt{(Csc^2x + Cot^2x)^2}}}$$

$$Q = \frac{\sqrt{1+Sec^2x + Tan^2x}}{\sqrt{1+Csc^2x + Cot^2x}}$$

$$Q = \frac{\sqrt{2Sec^2x}}{\sqrt{2Csc^2x}}$$

$$Q = \frac{|Secx|}{|Cscx|}$$

$$Q = |Tanx|$$

$\therefore$ **Q = Tanx**

CLAVE: A

IDENTIDADES TRIGONOMÉTRICAS FUNDAMENTALES

12. Reducir : $P = \sqrt{\dfrac{(1+\text{Tan}\theta+\text{Sec}\theta)(1+\text{Cot}\theta+\text{Csc}\theta)}{1+\text{Tan}\theta+\text{Cot}\theta+\text{Sec}\theta+\text{Csc}\theta}}$

RESOLUCIÓN:

$$P = \sqrt{\frac{(1+\text{Tan}\theta+\text{Sec}\theta)(1+\text{Cot}\theta+\text{Csc}\theta)}{1+\text{Tan}\theta+\text{Cot}\theta+\text{Sec}\theta+\text{Csc}\theta}}$$

$$P = \sqrt{\frac{1+\text{Cot}\theta+\text{Csc}\theta+\text{Tan}\theta+1+\text{Sec}\theta+\text{Sec}\theta+\text{Csc}\theta+\text{Sec}\theta\text{Csc}\theta}{1+\text{Sec}\theta\text{Csc}\theta+\text{Sec}\theta+\text{Csc}\theta}}$$

$$P = \sqrt{\frac{2+2\text{Sec}\theta+2\text{Csc}\theta+2\text{Sec}\theta\text{Csc}\theta}{1+\text{Sec}\theta\text{Csc}\theta+\text{Sec}\theta+\text{Csc}\theta}}$$

$\therefore \mathbf{P = \sqrt{2}}$

CLAVE: A

IDENTIDADES TRIGONOMÉTRICAS FUNDAMENTALES

13. Reducir la expresión : $P = (2 + \text{Tan}^2x)\text{Secx}.\,\text{Cot}^3x + \text{Cscx}$

RESOLUCIÓN:

$$P = (2 + \text{Tan}^2x)\text{Secx}.\,\text{Cot}^3x + \text{Cscx}$$

$$P = (2 + \text{Tan}^2x)\underbrace{\text{SecxCotx}}_{\text{Cscx}}\text{Cot}^2x + \text{Cscx}$$

$$P = \text{Cscx}[(2 + \text{Tan}^2x)\text{Cot}^2x + 1]$$

$$P = \text{Cscx}[(2\text{Cot}^2x + 1) + 1]$$

$$P = 2\text{Cscx}[1 + \text{Cot}^2x]$$

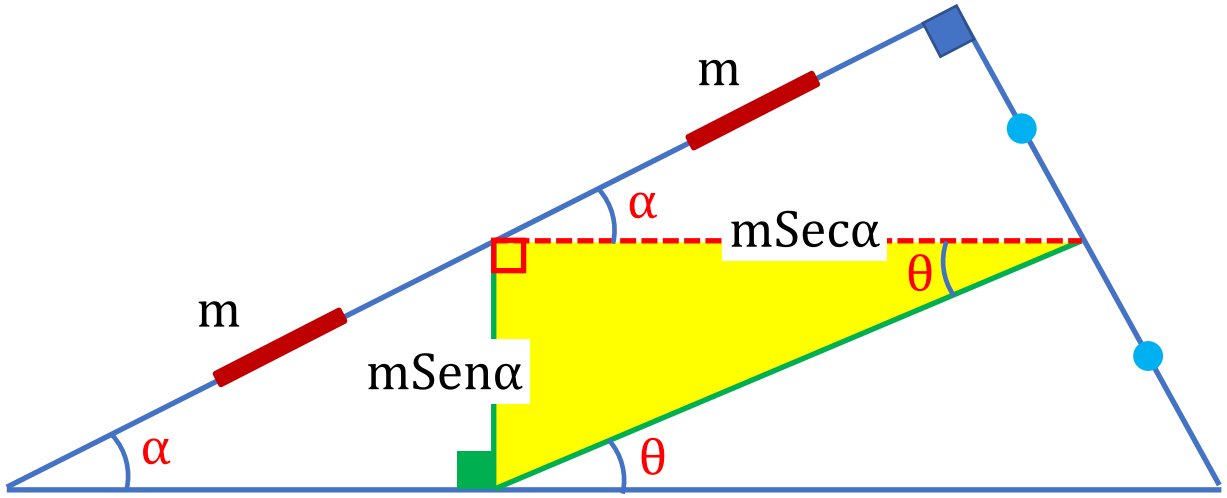
$$P = 2\text{Cscx}[\text{Csc}^2x]$$

$$\therefore \mathbf{P = 2\text{Csc}^3x}$$

CLAVE: E

IDENTIDADES TRIGONOMÉTRICAS FUNDAMENTALES

14. De la figura, a qué es igual : $M = (Tan\alpha + Cot\alpha)Tan\theta$



RESOLUCIÓN:

$$M = Sec\alpha Csc\alpha . Tan\theta$$

$$M = \cancel{Sec\alpha} Csc\alpha \times \frac{\cancel{m}Sen\alpha}{\cancel{mSec\alpha}}$$

$$\therefore \mathbf{M = 1}$$

CLAVE: A

IDENTIDADES TRIGONOMÉTRICAS FUNDAMENTALES

15. Siendo : $\text{Csc}^2x - \text{Cos}\theta = 1$, calcular : $M = \dfrac{\text{Tan}^4x - \text{Tan}^2\theta}{\text{Cot}^4x + \text{Sen}^2\theta}$

RESOLUCIÓN:

$$\text{Csc}^2x - \text{Cos}\theta = 1$$

$$1 + \text{Cot}^2x - \text{Cos}\theta = 1$$

$$\text{Cot}^2x = \text{Cos}\theta$$

AL^{-1}: $\text{Tan}^2x = \text{Sec}\theta$

AL^2: $\text{Tan}^4x = \text{Sec}^2\theta$

AL^2: $\text{Cot}^4x = \text{Cos}^2\theta$

$$M = \frac{\text{Tan}^4x - \text{Tan}^2\theta}{\text{Cot}^4x + \text{Sen}^2\theta}$$

$$M = \frac{\text{Sec}^2\theta - \text{Tan}^2\theta}{\text{Cos}^2\theta + \text{Sen}^2\theta}$$

$$M = \frac{1}{1}$$

$$\therefore \mathbf{M = 1}$$

CLAVE: A

IDENTIDADES TRIGONOMÉTRICAS FUNDAMENTALES

16. Sabiendo que : $\text{Sen}^2x + \text{Sen}^2y = \frac{1}{3}$

hallar :

$$\text{Tan}^2x + \text{Tan}^2y + \frac{5}{2}\text{Tan}^2x \cdot \text{Tan}^2y$$

CLAVE: B

IDENTIDADES TRIGONOMÉTRICAS FUNDAMENTALES

17. Sabiendo que :

$$\text{Sen}\theta + \text{Tan}\theta + \text{Sec}\theta = a$$
$$\text{Cos}\theta + \text{Cot}\theta + \text{Csc}\theta = b$$

calcular “$\text{Tan}\theta$”

CLAVE: B

IDENTIDADES TRIGONOMÉTRICAS FUNDAMENTALES

18.Si :

$$\left(\frac{Tana}{Senx} - \frac{Tanb}{Tanx}\right)^2 = Tan^2a - Tan^2b$$

hallar "Cosx"

CLAVE: A

IDENTIDADES TRIGONOMÉTRICAS FUNDAMENTALES

19. Hallar M en términos de “a”, sabiendo que es independiente de “θ”

$$M = (a\text{Sen}\theta - b\text{Sen}^3\theta)^2 + (a\text{Cos}\theta - b\text{Cos}^3\theta)^2$$

CLAVE: A

IDENTIDADES TRIGONOMÉTRICAS FUNDAMENTALES

20. Si : $a\text{Csc}\Phi + a\text{Cot}\Phi = x$
$y\text{Sec}\Phi - y\text{Tan}\Phi = b$
hallar una relación entre a, b, x e y que sea independiente de "Φ"

CLAVE: E

FIN DE LA SESIÓN

PRACTICA Y APRENDERÁS